ORDINARY LEVEL NATIONAL EXAMINATIONS 2012

SUBJECT: MATHEMATICS 1

DURATION: 3 HOURS

INSTRUCTIONS:

- This paper has TWO sections A and B.

  SECTION A: Answer ALL questions. (55 marks)

  SECTION B: Answer any THREE questions. (45 marks)

- You may use mathematical instruments and calculators where necessary.

- USE A BLUE INK PEN ONLY

- USE A PENCIL TO DRAW DIAGRAMS.

- SHOW CLEARLY ALL THE WORKING. Marks will not be awarded for answers without all working steps.
SECTION A: Attempt all questions. (55 marks)

1. Express 900 as a product of its prime factors. Hence find the square root of 900. (3 marks)
2. (a) Calculate without using a calculator: $3.45^2 - 1.55^2$. (2 marks)
   (b) Divide without using a calculator: $0.9 \div 30$. (1 mark)
3. In a school food store, there is enough food to feed 300 students for 17 days. For how long will the food last if 40 more students join the group? (3 marks)
4. Solve the equation: $5x^2 + 21x - 20 = 0$ (3 marks)
5. The right angled triangles below are similar. Find the area of the larger triangle. (3 marks)

![Triangles](image)

6. Solve the simultaneously: $x + 2y = 40$
   $3x = 60 - y$ (4 marks)
4. Find the equation of the line which passes through the points $(-1, 3)$ and $(4, 2)$. (4 marks)
5. Given that $f(x) = ax^2 - 7$ and $f(2) = 13$, find the value of $f(-1)$. (4 marks)
6. In a class of 40 students, 24 like Mathematics and 30 like Kinyarwanda. All students like at least one of the subjects. Draw a Venn diagram to represent this information. How many students like both mathematics and Kinyarwanda? (4 marks)
10. Solve the equation: $\frac{3x}{2} \geq \frac{x}{4} - 10$. Illustrate the answer on a number line. (4 marks)
11. A point $m$ divides a line segment $AB$, 10 cm long into two parts such that one part is 4 cm longer than the other. Find the length of the two parts. (4 marks)
12. The diagrams below show a flag $T$ and two mirrors $m_1$ and $M_2 M_1$ ($T$) in intersecting at an angle $Y^\circ$. Copy the diagram and show images $M_1 (T)$ in $m_1$ and $M_2 M_1$ ($T$) in $m_2$. (4 marks)
13. Given $152n = 68ten$, find $n$. 

14. Find the mid-point $M$ of the line joining the points $A (1, 0)$ and $B (9, 6)$. Find length $\overline{MB}$. 

![Diagram of a line segment with coordinates](image)

If it is curved in such a way that $MP$ and $NO$ meet to form a hollow cylindrical figure, find the volume of the cylindrical figure formed. $\pi = \frac{22}{7}$. 

SECTION B: Attempt ONLY three questions. (45 marks)

16. (a) Solve for $x$: $\frac{1}{x^2-1} + \frac{1}{x^2-4x+3} + \frac{1}{x-3} = 0$ 

(b) Factorize completely: $f(x) = 2x^3 + 5x^2 + x - 2$. Hence find the values of $x$ when $f(x) = 0$ 

17. The table below shows the ages of 73 students.

<table>
<thead>
<tr>
<th>Age in years</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>9</td>
<td>13</td>
<td>11</td>
<td>12</td>
<td>15</td>
<td>8</td>
</tr>
</tbody>
</table>

Make the frequency table using the above data.

(a) Find the median age. 
(b) Calculate the mean age. 

18. Use the diagram to answer (a) and (b) below.
(a) Copy the diagram and sketch the image under
   (i) a + 90° rotation about origin.          (3 marks)
   (ii) a - 180° rotation about origin.       (2 marks)

(b) Copy the diagram again and sketch the image of _______ under a translation. (3 marks)

   (i) $T = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$. (2 marks)
   (ii) $T = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$. (5 marks)

19. The figure below is a right trapezoidal prism.

![Diagram of trapezoidal prism]

Calculate its (a) lateral area. (5 marks)
(b) total surface area. (7 marks)
(c) volume. (3 marks)

20. (a) Rationalize the denominator: $\frac{\sqrt{2}}{2\sqrt{5} + \sqrt{3}}$. (4 marks)

(b) Simplify: $\sqrt{12} \times 3 \sqrt{60} \times \sqrt{45}$. (4 marks)

(c) Simplify: $\sqrt{8} \times \sqrt{50} + \sqrt{121}$. (4 marks)

(d) Simplify: $\frac{5\sqrt{7}}{\sqrt{45}} \times \frac{2\sqrt{3}}{\sqrt{21}}$. (3 marks)

END.
### ANSWERS FOR NATIONAL EXAMINATION 2012.
**MATHEMATICS 009**
**SECTION A**

<table>
<thead>
<tr>
<th>1.</th>
<th>2. a) $45^2 - 1.55^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>$(3.45+1.55)(3.45-1.55)$</td>
</tr>
<tr>
<td>3.</td>
<td>$(5.00)(1.90)$</td>
</tr>
<tr>
<td>4.</td>
<td>So $900 = 2^2 \times 3^2 \times 5^2$</td>
</tr>
<tr>
<td>5.</td>
<td>$\sqrt{900} = \sqrt{2^2 \times 3^2 \times 5^2}$</td>
</tr>
<tr>
<td>6.</td>
<td>$= 2 \times 3 \times 5 = 30$</td>
</tr>
</tbody>
</table>

| 2. b) $0.9 \div 30 = \frac{9}{10} \times \frac{1}{30}$ |
| - | $= \frac{3}{100 \times 10} = 0.003$ |

| 3. | 300 students $\Rightarrow$ 17 days |
| - | 1 student $\Rightarrow 17 \times 300$ |
| 4. | $340$ students $\Rightarrow \frac{17 \text{ days} \times 300}{340}$ |
| - | $= 15$ days. |

| 4. | $5x^2 + 21x - 20 = 0$ |
| - | $= 5x^2 + 25x - 4x - 20 = 0$ |
| 5. | $= 5(x+5) - 4(x+5) = 0$ |
| 6. | $= (x+5)(5x-4) = 0$ |
| 7. | $\therefore x + 5 = 0$ or $5x - 4 = 0$ |
| 8. | $x = -5$ or $x = \frac{4}{5}$ |

| 5. | Area of the small triangle |
| - | $\frac{1}{2} \times 4 \times 3 = 6\text{cm}^2$ |
| 6. | Linear scale factor $= \frac{20}{5} = 4$ |
| 7. | Area scale factor $= 4^2 = 16$ |
| 8. | Area of larger triangle |
| - | $= 16 \times 6 = 96\text{cm}^2$ |

| 6. | $\times 3 \quad x + 2y = 40$ |
| - | $3x + y = 60$ |
| 7. | $= 3x + 6y = 120$ |
| 8. | $3x + y = 60$ |
| 9. | $5y = 60$ |
| 10. | $y = 12$ |
| 11. | $x + 24 = 40$ |
| 12. | $x = 16$ |

| 7. | Let $x$ and $y$ be any point on the line. |
| - | Then the gradient of the line |
| 8. | $\frac{y-3}{x-(-1)} = \frac{y-3}{x+1}$ |
| 9. | Again the gradient of the line $= \frac{2-3}{4-(-1)} = \frac{-1}{5}$ |
| 10. | So $\frac{y-3}{x+1} = \frac{-1}{5}$ |
| 11. | $\therefore 5(y-3) = -1(x+1) = 5y - 15 = -x - 1$ |
| 12. | $5y + x = 14$ |

| 8. | $f(x) = ax^2 - 7$ |
| - | $F(2) = a(2)^2 - 7 = 13$ |
| 9. | $\text{So } 4a - 7 = 13$ |
| 10. | $4a = 20$ |
| 11. | $a = 5$ |
| 12. | $\therefore f(x) = 5x^2 - 7$ |
| 13. | $F(-1) = 5(-1)^2 - 7$ |
| 14. | $= 5 - 7 = -2$ |
16. a) \[
\frac{1}{x^2-1} + \frac{1}{x^2-4x+3} + \frac{1}{x-3} = 0
\]
\[
= \frac{1}{(x-1)(x+1)} + \frac{1}{(x-3)(x-1)} + \frac{1}{x-3} = 0
\]
\[
= x - 3 + x + 1 + (x - 1)(x + 1) = 0
\]
\[
= x^2 + 2x - 3 = 0
\]
\[
= x^2 + 3x - x - 3 = 0
\]
\[
= x(x + 3) - (x + 3) = 0
\]
\[
= (x + 3)(x - 1) = 0
\]
\[
x = -3
\]
(x = 1 is not valid)

b) \[
2x^3 + 5x^2 + x - 2
\]
\[
= 2x^3 - x^2
\]
\[
= 6x^2 + x
\]
\[
= 6x^2 - 3x
\]
\[
= 4x - 2
\]
\[
= 4x - 2
\]
\[
= 0
\]
\[
x^2 + 3x + 2 = x^2 + 2x + x + 2
\]
\[
= x(x + 2) + (x + 2)
\]
\[
= (x + 2)(x + 1)
\]
So \[
2x^3 + 5x^2 + x - 2 = (2x - 1)(x + 2)(x + 1)
\]
\[
x = \frac{-2}{2} \text{ or } x = -1 \text{ or } x = -\frac{1}{2}
\]

17.

<table>
<thead>
<tr>
<th>Age in years</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>9</td>
<td>13</td>
<td>11</td>
<td>12</td>
<td>15</td>
<td>8</td>
</tr>
<tr>
<td>Cumulative frequency</td>
<td>5</td>
<td>14</td>
<td>27</td>
<td>38</td>
<td>50</td>
<td>65</td>
<td>73</td>
</tr>
</tbody>
</table>

a) Median age = the \(\frac{1}{2}(N + 1)\)th age

\[= \frac{1}{2}(73 + 1)\]

median age = the 37th age.

b) Mean age = \(\frac{1261}{73} = 17.27\)
18.

a)

(i) 

(ii) 

b)

(i) 

(ii) 

\[ T = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \]

\[ T = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \]
19.

a) Lateral area
   \[ \text{base perimeter} \times \text{height} \]
   \[ = (5+6+5+12) \times 10 \]
   \[ = 28 \times 10 = 280 \text{cm}^2 \]

b) Total surface area
   \[ = \text{lateral area} + \text{area of 2 bases} \]
   \[ = \text{base area} = \frac{1}{2} \times 4 \times (6+12) \]
   \[ = 36 \text{cm}^2 \]
   Area of 2 bases = \(2 \times 36 = 72 \text{cm}^2\)
   Total area = 280 + 72 = 352cm²

c) Volume
   \[ = \text{base area} \times \text{height} \]
   \[ = 36 \times 10 \]
   \[ = 360 \text{cm}^3 \]

20.

a) \[ \frac{\sqrt{2}}{2\sqrt{5} + \sqrt{3}} = \frac{\sqrt{2}(2\sqrt{5} - \sqrt{3})}{(2\sqrt{5} + \sqrt{3})(2\sqrt{5} - \sqrt{3})} = \frac{2\sqrt{10} - \sqrt{6}}{4\times 5 - 3} \]
   \[ = \frac{2\sqrt{10} - \sqrt{6}}{17} \]

b) \[ \sqrt{12} \times 3 \sqrt{60} \times \sqrt{45} = \sqrt{4 \times 3 \times 3 \times 4 \times 15} \times \sqrt{9 \times 5} \]
   \[ = 2\sqrt{3} \times 6\sqrt{15} \times 3\sqrt{5} = 36\sqrt{3} \times 15 \times 5 = 36 \times 15 \]
   \[ = 36 \times 15 \]
   \[ = 540 \]

c) \[ \sqrt{8} \times \sqrt{50} + \sqrt{121} \]
   \[ = \sqrt{4 \times 2} \times \sqrt{25 \times 2} + \sqrt{11 \times 11} \]
   \[ = 2\sqrt{2} \times 5\sqrt{2} + 11 \]
   \[ = 10\sqrt{2} \times 2 + 11 \]
   \[ = 10 \times 2 + 11 \]
   \[ = 31 \]

d) \[ \frac{5\sqrt{7}}{\sqrt{45}} \times \frac{2\sqrt{3}}{\sqrt{21}} = \frac{5\sqrt{7} \times 2\sqrt{3}}{\sqrt{9 \times 7 \times 3}} = \frac{5 \times 2}{3\sqrt{5}} \]
   \[ = \frac{10\sqrt{5}}{3\sqrt{5}} \]
   \[ = \frac{2\sqrt{5}}{3} \]

END.