NATIONAL EXAMINATION 2005

SUJECTS : MATHEMATICS VII

LEVEL : TRONC COMMUN

DURATION : 3 HOURS

INSTRUCTIONS:

- This paper consists of **TWO** Sections A and B.
- Attempt **ALL** questions in Section A and any **THREE** questions in Section B.
- Show **ALL** working clearly.
- Calculators and mathematical instruments may be used.
SECTION A

1. Simplify: \((2\frac{1}{2} + 7\frac{1}{2}) + \frac{1}{4}\). (3 marks)

2. Given that \(f(x) = 2(x - 1)\), find
   a) \(f(-2)\) (1 mark)
   b) the value of \(x\) such that \(f(x) = -8\). (2 marks)

3. Simplify the following completely: \(\frac{3(m-y) - 2(m-3y)}{m^2 - 9y^2}\). (3 marks)

7. At a certain factory, the salary of a worker for 40 hours per week is 20,000 Rwf. The worker is paid 10% of the weekly salary for every 2 hours that he works overtime. If at the end of a certain week he received 36,000 Rwf, find the number of hours he worked overtime. (3 marks)

8. Solve the inequality: \(2y - \frac{7(y-1)}{2} > 17\). (3 marks)

9. It is given that \(\vec{x} = \left(\frac{4}{3}\right)\) and \(\vec{y} = \left(\frac{12}{9}\right)\). Find: (a) the column vector of \(\vec{y} - \vec{x}\). (1 mark)
   (b) the value of \(|\vec{x}| + |\vec{y}|\). (3 marks)

7. The equation of line A is \(2y = 3x + 4\). Line B passes through points \((1, 0)\) and \((a, 2)\). If A is parallel to line B, find the value of \(a\). (4 marks)

8. In the diagram below show that triangle CDE is similar to triangle CAB. (2 marks)

9. Muṣa, Sarah and Peter, share 1000 kg of sugar in the ratio 2: x: 5 respectively. If Musa gets 200 kg of sugar, how much sugar does Sarah get? (4 marks)

10. Solve \((x + 3)(x - 4) = 18\). (4 marks)
11. In the figure below find the value of y. (4 marks)

![Triangle Diagram]

12. (a) Copy the table below and complete it. (3 marks)

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Find the equation connecting x and y. (1 mark)

13. Determine the area enclosed by the lines x = 3, y = -1 and x + y = 5. (4 marks)

14. The pie chart below shows the drinks popularly liked by students in a certain school.

![Pie Chart]

If 60 students like sprite, determine

(a) the number of students who like each type of drink. (3 marks)

(b) the total number of students in the school. (1 mark)

15. A (3, 0) is a point in a Cartesian plane.

If A (3, 0) is mapped on to point B (5, 2) by a translation, find the image of C (2, -3) under the same translation. (2 marks)
SECTION B: Answer any THREE questions (45 Marks)

16. (a) Solve simultaneously: \(x + y = 3\)
\[2x^2 + y^2 + 3x = 15.\] (10 marks)

(b) Use the answer from 16 (a) and solve: \[\frac{m\, y - 3m^2 + 2}{4 - y} = 0.\] (5 marks)

17. The table below shows the ages of 31 secondary school students.

<table>
<thead>
<tr>
<th>Ages (x)</th>
<th>Frequency (f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>17</td>
<td>19</td>
</tr>
<tr>
<td>18</td>
<td>14</td>
</tr>
<tr>
<td>19</td>
<td>14</td>
</tr>
<tr>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td>20</td>
<td>14</td>
</tr>
</tbody>
</table>

a) Copy the table below and complete it.

<table>
<thead>
<tr>
<th>Ages (x)</th>
<th>Frequency (f)</th>
<th>(F(x))</th>
<th>Commutative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[\sum f = \quad \sum fx = \quad \] (11½ marks)

b) State the mode. (1 mark)

c) Find the median age. (2½ marks)

d) Calculate the average age. (2½ marks)

18. (a) Solve: \(2x^3 - 5x^2 - 9x + 18 = 0, x \in \mathbb{R}\). (10 marks)

(b) Factorize and simplify completely: \[\frac{x^4 - 6x^3 + 9x^2}{x^2 - 3x^3}.\] (5 marks)

19. The distance between two schools G and K is 432km. A bus transporting students leaves school G at 7.00 am for school K. The bus travels at an average speed of 72 km/hr. One and a half hours later, a mini bus leaves school G at a steady speed of 108km/hr heading for school K. All the two vehicles travel nonstop until they arrive in school k.
(a) Calculate the times when the two vehicles arrive in school K.  
(b) On a graph and on the same axes show the journey of the two vehicles.

Use your graph to estimate the:

i) The time and distance from school G when the minibus overtakes the bus.  
ii) The distance at 10.00 am between the two vehicles.  
iii) The time when the bus is 16 km behind the minibus.

20. A sample of 100 men was taken to find out how many were literate (L), married (M) or had an income generating project (P). The following were the findings: n (L) = 47, n (M) = 59, n (P) = 52, n (L∩M) = 30, n (L∩P) = 24 and n (M∩P) = 34. 14 of the men were literate, unmarried and without an income generating project.

(a) Represent this information on a Venn diagram.  
(b) Determine the number of men who were

(i) Literate, married and had an income generating project.
(ii) Neither literate nor married.

END
**ANSWERS FOR NATIONAL EXAMINATION 2005.**
**MATHEMATICS VI**
**SECTION A**

1. \((2\frac{1}{2} + 7\frac{1}{2}) + \frac{1}{4}\)  
   \[= \left(\frac{5}{2} + \frac{15}{2}\right) + \frac{1}{4}\]  
   \[= \frac{5}{2} \times \frac{2}{15} + \frac{1}{4} = \frac{10}{30} + \frac{1}{4}\]  
   \[= \frac{4 + 3}{12}\]  
   \[= \frac{7}{12}\]

2. a) \(f(-2) = 2(-2-1)\)  
   \[= 2(-3)\]  
   \[= -6\]  

b) \(2(x-1) = -8\)  
   \[2x - 2 = -8\]  
   \[2x = -8 + 2\]  
   \[2x = -6\]  
   \[x = -3\]

3. \[\frac{3(m-y) - 2(m-3y)}{m^2 - 9y^2}\]  
   \[= \frac{3m - 3y - 2m + 6y}{(m - 3y)(m + 3y)}\]  
   \[= \frac{m + 2y}{m - 3y}\]  
   \[= \frac{1}{m - 3y}\]

4. 2 hours over time:  
   \[\frac{10}{100} \times 20,000 = 2000\]  

Money he got over time:  
\[= 36,000 - 20,000 = 16000\text{Fr}\]  

2 hrs = 2000Rwf  
1 hour = 1000Rwf  
\(X\) hrs = \[\frac{16000}{1000} = 10\text{ hrs}\]  
\[\therefore\text{He worked for 16 hrs over time}\]

5. \[2y - \frac{7(y-1)}{2} > 17\]  
   \[= 2y - 7y + 7 > 34\]  
   \[= -5y > 27\]  
   \[= -y > 5.4\]  
   \[= y < -5.4\]

6. a) \(\ddot{y} - \ddot{x} = \left(\frac{12}{6} - \frac{4}{6}\right) = \left(\frac{8}{6}\right)\)

b) \(|\ddot{x}| + |\ddot{y}|\)  
   \[= \sqrt{4^2 + 3^2} + \sqrt{12^2 + 9^2}\]  
   \[= \sqrt{16 + 9} + \sqrt{144 + 81}\]  
   \[= \sqrt{25} + \sqrt{225}\]  
   \[= 5 + 15\]  
   \[= 20\]
7. B stands for \((1, 0)\) and \((a, 2)\)

\[
\begin{align*}
M &= \frac{3}{2} \quad A &= \frac{3}{2} \\
4a &= 3a - 3 \\
4 + 3 &= 3a \\
a &= \frac{7}{3}
\end{align*}
\]

8. Angle in \(C\), from \(CDE\) that \(CAB\), has an amplitude two opposing sides as the top \(C\).

As \(ED//AB\):

Angle in \(E\) = angle in \(B\)

(alternate angles)

Also angle in \(D\) = angle in \(A\)

(alternate angle)

Hence, the correspondent angles have the same amplitude.

9. Let \(T\) be the total ratio

\[
\begin{align*}
\text{Musa's share} &= \frac{2}{T} \times 1000 = 200 \\
\frac{2000}{T} &= 200 \\
\text{Total ratio} &= 10 \\
2 + x + 5 &= 10 \\
x &= 3 \\
\text{Sarah's share} &= \frac{3}{10} \times 1000 \\
&= 300\text{kg}
\end{align*}
\]

10. \((x + 3) (x - 4) = 18\)

\[
\begin{align*}
(x + 3) (x - 4) &= 18 \\
x^2 - x - 12 - 18 &= 0 \\
x^2 - x - 30 &= 0 \\
x^2 - 6x + 5x - 30 &= 0 \\
(x^2 - 6x) + (5x - 30) &= 0 \\
x(x - 6) + 5(x - 6) &= 0 \\
(x + 5) (x - 6) &= 0 \\
x + 5 &= 0 \text{ or } x - 6 = 0 \\
x &= -5 \quad x = 6
\end{align*}
\]

11. 

\[
\begin{align*}
(x + 3) (x - 4) &= 18 \\
(x + 3) (x - 4) &= x + x + 10 + 3x + 80 \\
(x + 3) (x - 4) &= 180 \\
x^2 - x - 12 - 18 &= 0 \\
5x + 90 &= 180 \\
x &= 18 \\
x &= x + 18 \\
y + 80 + x + 10 &= 180 \\
y + 80 + 18 &= 180 \\
y &= 180 - 98 \\
y &= 82°
\end{align*}
\]

12. a)

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
X & 1 & 2 & 4 & 8 & 16 & 20 \\
\hline
Y & 23 & 11 & 5 & 4 & 2 & 1 \\
\hline
\end{array}
\]

b) \(m = \frac{1 - 5}{20 - 4} = \frac{-4}{16} = \frac{-1}{4}\)

\[
y - a_2 = m(x - a_1) \Rightarrow y - 5 = \frac{-1}{4} (x - 4)
\]

\[
y - 5 = \frac{-1x}{4} + 1 + 5
\]

\[
y = \frac{-1x}{4} + 1 + 5
\]

\[
y = \frac{-1}{4}x + 6
\]
13. Area = \( \frac{3 \times 3}{2} = 4.5 \) 

14. Total number of students: 
\[ \frac{40}{360} \times 100 = 60 \] 
\[ \frac{40}{360} x \times 100 \times \frac{21600}{40} = 540 \] 
students

14. continuation: 
- Milk = \( \frac{150}{360} \times 540 = 225 \) students
- Juice = \( \frac{90}{360} \times 540 = 135 \) students
- Coca Cola = \( \frac{80}{360} \times 540 = 120 \) students

b) Total number of students: 
\[ = 225+60+135+120 = 540 \]

15. 
- B \((5, 2)\) image by translation 
\[ \begin{bmatrix} 5 \end{bmatrix} - \begin{bmatrix} 2 \end{bmatrix} = \begin{bmatrix} 2 \end{bmatrix} \] 
\[ t = \begin{bmatrix} 2 \end{bmatrix} \]
- For \( t = \begin{bmatrix} 2 \end{bmatrix} \) c \((2, -3)\) will have the image \( c^1 \((4, -1)\)

SECTION B

16. a) \( x + y = 3 \) 
- \( 2x^2 + y^2 + 3x = 15 \) 
- \( x = 3 - y \) 
- \( 2(3-y)^2 + y^2 + 3(3-y) = 15 \) 
- \( 2(9-6y+y^2) + y^2 + 9-3y = 15 \) 
- \( 18-12y +2y^2+y^2 + 9-3y = 15 \)
- using equation \( x = 3 - y \), 
- \( x = 3 - 4 = -1 \)

b) \( 3y^2-15y+27=15 \) 
- \( 3y^2-15y+27-15=0 \) 
- \( 3y^2 + 5y + 4 = 0 \) 
- \( 3(y-1)(y+4) = 0 \) 
- \( y-4 = 0 \) 
- \( y = 4 \)
- \( y = 4 \)
17. 

<table>
<thead>
<tr>
<th>Ages (x)</th>
<th>Frequency (f)</th>
<th>Fx</th>
<th>Commutative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>7</td>
<td>98</td>
<td>7</td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>60</td>
<td>11</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>32</td>
<td>13</td>
</tr>
<tr>
<td>17</td>
<td>5</td>
<td>85</td>
<td>18</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td>54</td>
<td>21</td>
</tr>
<tr>
<td>19</td>
<td>5</td>
<td>95</td>
<td>26</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>100</td>
<td>31</td>
</tr>
</tbody>
</table>

\[ \sum f = 31 \quad \sum fx = 524 \]

b) mode = 14

c) median = \( \frac{20+14}{2} = 17 \)

d) average age = \( \frac{524}{31} = 17 \) years

18. a) \( 2x^3 - 5x^2 - 9x + 18 = 0 \)

\[
(x + 2 \text{ is a factor})
\]

\[
= 2(-2) - 5(-2)^2 - 4(-2) + 18
\]

\[
= -10 - 20 + 18 + 18
\]

\[
= 36 + 36 = 0
\]

\[
\begin{array}{ccc|c}
2 & -5 & -9 & 18 \\
\hline
-4 & 18 & -18 \\
\hline
-2 & & & \\
\hline
2 & -4 & 9 & 0 \\
\end{array}
\]

\[
2x^2 - 4x + 4 = 0
\]

\[
2x - 6x - 3x + 4 = 0
\]

\[
(2x^2 - 6x) + -3x + 4 = 0
\]

\[
2x(x - 3) -3(x - 3) = 0
\]

\[
(2x - 3) (x - 3) 0
\]

\[
2x - 3 = 0
\]

\[
x = \frac{3}{2} \quad \text{or} \quad x = 3
\]

b) \[
\frac{x^4 - 6x^3 + 9x^2}{x^3 - 3x^2}
\]

\[
\frac{x^2(x^2 -6x+9)}{x^2(x-3)}
\]

\[
\frac{x^2-6x+9}{(x-3)}
\]

\[
(x -)(x - 3)
\]

\[
= x - 3
\]
19.

a) bus: $\frac{432}{72} = 6 \text{ hours}$

minibus: $\frac{432}{108} = 4 \text{ hours}$

b) i) Journey of the two vehicles on the graph paper (teacher's guidance)

The minibus overtakes the bus $\pm 312\text{km}$ from G to C between 11hrs 22 min to 4hrs 22 mins.

19. b) ii)

The distance at 10am is $\pm 50\text{km}$ between the 2 vehicles

iii) at 12 pm, the bus is 16km behind the minibus.

20.

a) $\frac{n(L) + n(M) + n(P)}{2} = 100$

$n(L) = 47$

$n(M) = 59$

$n(P) = 52$

b) i) $14x + x + 30 - x + 24 - x = 47$

$x = 21$

Number of $(L \cap M \cap P) = 21$

ii) $n(L \cap M)^1$

$n(L \cap P)\text{ only} = 24 - x$

$= 24 - 21$

$= 3 \text{ men}$

$n(M \cap P)\text{ only} = 34 - x$

$= 34 - 21$

$= 13 \text{ men}$

$n(P)\text{ only} = 52 - (3 + x + 13)$

$= 52 - (16 + 21)$

$= 52 - 37$

$= 15 \text{ men}$

$\therefore 15 \text{ men were neither literate nor married.}$